WILLETTON SENIOR HIGH SCHOOL

Semester One Examination, 2022

Question/Answer booklet

MATHEMATICS APPLICATIONS UNIT 1

Section One: Calculator-free

SOLUTIONS

Your name _____

Teacher's name _____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section One: Calculator-free **35% (52 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 **(5 marks)**

(a) Evaluate $30 - 20 \div 2 + 3$. (1 mark)

Solution
$30 - 20 \div 2 + 3 = 30 - 10 + 3$ $= 23$
Specific behaviours
✓ correctly evaluates

(b) Determine the value of v when $t = 3$, $a = 2.5$ and $v = 2at^2 - (t + 5)(a - 2)$. (2 marks)

Solution
$v = 2(2.5)(3)^2 - (3 + 5)(2.5 - 2)$ $= 5(9) - 8(0.5)$ $= 45 - 4$ $= 41$
Specific behaviours
✓ substitutes ✓ evaluates

(c) Evaluate the expression $\frac{a + 16}{\sqrt{5a - b}}$ when $a = 8$ and $b = 4$. (2 marks)

Solution
$\frac{a + 16}{\sqrt{5a - b}} = \frac{8 + 16}{\sqrt{5(8) - 4}}$ $= \frac{24}{\sqrt{40 - 4}}$ $= \frac{24}{\sqrt{36}}$ $= 24 \div 6 = 4$
Specific behaviours
✓ substitutes ✓ evaluates

See next page

Question 2 **(5 marks)**

Aaron works part-time as a porter at a hotel. His timesheet for three shifts he worked in January this year is shown below.

Day	Start time	Finish time
Wednesday (Australia Day holiday)	$6{:}30\ \text{am}$	$1{:}00\ \text{pm}$
Thursday	$11{:}45\ \text{am}$	$7{:}45\ \text{pm}$
Friday	$10{:}15\ \text{am}$	$7{:}15\ \text{pm}$

The hotel pays porters $\$20$ per hour, except for (i) hours worked after 5 pm when time-and-a-half is paid; and (ii) all hours worked on a public holiday when double time is paid.

Determine Aaron's pay for the three shifts he worked.

Solution

Double time on Wed:

$$6\text{h } 30\text{m}$$

Time-and-a-half hours (after 5 pm) on Thurs and Fri:

$$2\text{h } 45\text{m} + 2\text{h } 15\text{m} = 5\text{h}$$

Normal hours (until 5 pm) on Thurs and Fri in hours and minutes:

$$5h\ 15\text{min} + 6h\ 45\text{min} = 12\text{h}$$

Using bonus rates, time paid, in hours:

$$12 + (5 \times 1.5) + (6.5 \times 2) = 12 + 7.5 + 13 = 32.5$$

Aaron's pay for three shifts:

$$32.5 \times 20 = \$650$$

NB Alternative methods exist, such as pay per day resulting in Wed: $\$260$*, Thurs:* $\$187.50$*, Fri:* $\$202.50$*, totalling* $\$650$*.*

Specific behaviours

✓ indicates hours worked at normal rates
✓ indicates hours worked at time-and-a-half
✓ indicates hours worked at double time
✓ correctly adjusts time worked by bonus rates
✓ calculates total pay

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Question 3 **(8 marks)**

(a) State the size of the following matrices in the form $a \times b$.

(i) $\begin{bmatrix} 1 & 0 & -2 & 1 \\ -2 & 1 & 0 & 3 \end{bmatrix}$ (1 mark)

Solution
Size is 2×4.
Specific behaviours
✓ correct size in required form

(ii) The identity matrix with 5 columns. (1 mark)

Solution
Size is 5×5.
Specific behaviours
✓ correct size in required form

(b) The row matrix $\mathbf{Z}$ has 3 elements. Write a suitable matrix $\mathbf{Z}$, given that its 3 elements are all whole numbers and have a sum of 7. (2 marks)

Solution
One of many possibilities is $\mathbf{Z} = [1 \quad 2 \quad 4]$.
Specific behaviours
✓ correct size matrix ✓ elements are non-negative integers with correct sum

(c) Write as a single matrix

(i) $\begin{bmatrix} 0 & -2 \\ 5 & 3 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$. (2 marks)

Solution
$\begin{bmatrix} 0 & -2 \\ 5 & 3 \end{bmatrix} + 3\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 3 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ -3 & 0 \end{bmatrix}$ $= \begin{bmatrix} 6 & 1 \\ 2 & 3 \end{bmatrix}$
Specific behaviours
✓ correctly calculates scalar multiple ✓ correct single matrix

(ii) $\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix}$. (2 marks)

Solution
$\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 8+6 \\ -2-9 \end{bmatrix} = \begin{bmatrix} 14 \\ -11 \end{bmatrix}$
Specific behaviours
✓ 2×1 matrix with one element correct ✓ correct single matrix

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Question 4 **(9 marks)**

Consider the sequences $7, 9, 11, 13, \ldots$ and $7, 17, 27, 37, \ldots$. They both start at 7 but the numbers in the first sequence go up in 2's and the numbers in the second go up in 10's.

(a) Calculate $7 + 10 + 13 + 16 + 19$. (1 mark)

Solution
$7 + 10 + 13 + 16 + 19 = 65$
Specific behaviours
✓ correct sum

The formula $S = \frac{1}{2}\big(n(d(n-1)+14)\big)$ can be used to determine the sum S of the first n numbers in sequences that start at 7 and go up by d each time.

(b) Show use of the formula to determine the value of S when $n = 5$ and $d = 3$, and explain why this value is the same as your answer to part (a). (3 marks)

Solution
$S = \frac{1}{2}\big(5(3(5-1)+14)\big)$ $= \frac{1}{2}\big(5(12+14)\big)$ $= 5 \times 13 = 65$ This is the same as (a) since the sequence in (a) has 5 numbers ($n = 5$) starting at 7 and going up in 3's ($d = 3$).
Specific behaviours
✓ substitutes correctly ✓ shows simplification before reaching correct result ✓ explains using number of terms and step

This spreadsheet uses the above formula to calculate the value of S for some values of n and d.

		n										
		20	21	22	23	24	25	26	27	28	29	30
d	3	710	777	847	920	996	1075	1157	1242	1330	1421	1515
	4	900	987	1078	1173	1272	1375	1482	1593	1708	1827	1950
	5	1090	1197	1309	1426	1548	1675	1807	1944	2086	2233	2385
	6	1280	1407	1540	1679	1824	1975	2132	2295	2464	2639	2820
	7	1470	1617	1771	1932	2100	2275	2457	2646	2842	3045	3255
	8	1660	1827	2002	2185	2376	2575	2782	2997	3220	3451	3690

(c) State the value of S when $n = 29$ and $d = 5$. (1 mark)

Solution
$S = 2233$
Specific behaviours
✓ correct value of S

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(d) Determine the sum of the first 27 numbers in the sequence that starts $7, 14, 21, 28, \ldots$. (2 marks)

Solution
$n = 27, \quad d = 7$ Hence $S = 2646$ from spreadsheet.
Specific behaviours
✓ indicates correct values of n and d or substitution and some correct simplification in formula ✓ correct value of S

Consider another formula $T = \frac{n^2 d}{2} - \frac{nd}{2} + 7n$.

A claim was made that S and T will always be equal for any given value of d and value of n.

(e) Show that when $n = 30$ and $d = 6$ the claim is true. (2 marks)

Solution
$T = \frac{30^2 \times 6}{2} - \frac{30 \times 6}{2} + 7 \times 30$ $= 900 \times 3 - 90 + 210$ $= 2820$ This is the same as the value of S shown in the spreadsheet, or calculated using the formula for S $S = \frac{1}{2}\big(30(6(30-1)+14)\big)$ $= 2820$
Specific behaviours
✓ Correct T value showing simplification ✓ stating/calculating correct S value and noting/stating $T = S$

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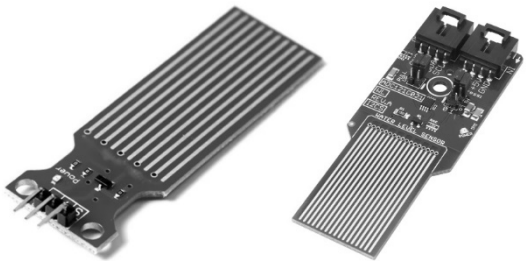
Question 5 **(10 marks)**

A company manufactures Standard and Premium liquid level sensors (shown at right), with each model requiring different numbers of capacitors, diodes, resistors and transistors as shown in matrix $\mathbf{M}$:



$$\mathbf{M} = \begin{bmatrix} 2 & 3 & 7 & 1 \\ 1 & 5 & 6 & 3 \end{bmatrix}$$

The numbers in the first row of $\mathbf{M}$ are the components required for the Standard model, and the numbers in the second row are the components for the Premium model. The first, second, third and fourth columns of matrix $\mathbf{M}$, show the number of capacitors, diodes, resistors and transistors respectively.

(a) Write down the element $m_{1,2}$ of $\mathbf{M}$ and describe what this number represents. (2 marks)

Solution
$m_{1,2} = 3$. This number represents the number of diodes required for a Standard model.
Specific behaviours
✓ correct element ✓ identifies correct component and model

The cost in cents, of capacitors, diodes, resistors and transistors from Supplier A is $20, 20, 10$ and 50 respectively, whilst from Supplier B is $15, 30, 10$ and 40 respectively.

(b) Arrange this information into cost matrix $\mathbf{C}$ and hence determine matrix $\mathbf{R}$, where $\mathbf{R} = \mathbf{M} \times \mathbf{C}$.

(3 marks)

Solution
$\mathbf{C} = \begin{bmatrix} 20 & 15 \\ 20 & 30 \\ 10 & 10 \\ 50 & 40 \end{bmatrix}$, $\mathbf{R} = \mathbf{M} \times \mathbf{C} = \begin{bmatrix} 2 & 3 & 7 & 1 \\ 1 & 5 & 6 & 3 \end{bmatrix} \times \begin{bmatrix} 20 & 15 \\ 20 & 30 \\ 10 & 10 \\ 50 & 40 \end{bmatrix} = \begin{bmatrix} 220 & 230 \\ 330 & 345 \end{bmatrix}$
Specific behaviours
✓ correct cost 4X2 matrix (*columns can be swapped - follow through in c*) ✓ calculates at least one correct element of product in a 2X2 matrix ✓ correct product in a 2X2 matrix

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(c) Explain what the element $r_{2,1}$ of matrix $\mathbf{R}$ represents. (2 marks)

Solution
$r_{2,1}$ represents the total cost of \$3.30 for all components used in a Premium model from Supplier A.
Specific behaviours
✓ indicates total cost of \$3.30 for all components ✓ identifies correct model and supplier

(d) The company have an order for 200 Standard and 100 Premium models. Show use of matrix multiplication to calculate a 1×1 matrix that represents the total cost, in dollars, of the components required for this order from Supplier A. (3 marks)

Solution
$$[200 \quad 100] \times \begin{bmatrix} 2.20 \\ 3.30 \end{bmatrix} = [770]$$ or equivalent, such as... $$\frac{1}{100} \times [200 \quad 100] \times \begin{bmatrix} 220 \\ 330 \end{bmatrix} = [770]$$
Specific behaviours
✓ shows appropriate order and cost matrices ✓ shows use of matrix multiplication ✓ correct total cost matrix

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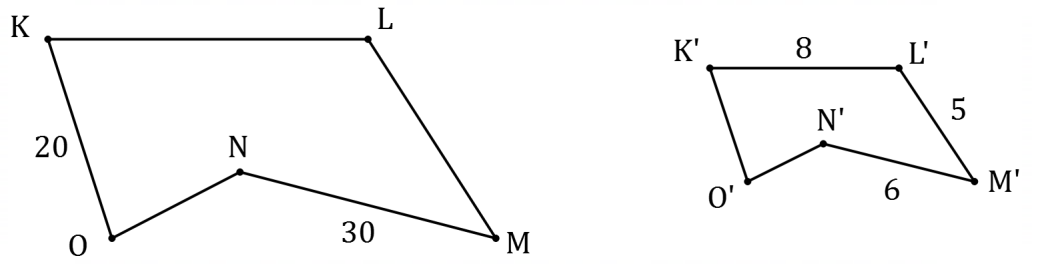
Question 6 **(7 marks)**

The diagram below, not drawn to scale, shows two similar pentagons $KLMNO$ and $K'L'M'N'O'$. The lengths of some of the sides are shown in centimetres.



(a) Show use of a scale factor to determine the length of side LM and the length of side $O'K'$. (3 marks)

Solution
Scale factor: $\frac{NM}{N'M'} = \frac{30}{6} = 5$ $LM = 5 \times 5 = 25$ cm, $\quad O'K' = 20 \div 5 = 4$ cm
Specific behaviours
✓ calculates scale factor (either 5 or $^1/_5$) ✓ length of LM ✓ length of $O'K'$

The perimeter of $KLMNO$ is 130 cm.

(b) Determine the length of side $O'N'$. (2 marks)

Solution	
$KL = 8 \times 5 = 40$ $ON = 130 - (20 + 40 + 25 + 30)$ $= 130 - 115 = 15$ $O'N' = 15 \div 5 = 3$ cm	$P_{K'L'M'N'O'} = 130 \div 5 = 26$ $O'N' = 26 - (8 + 5 + 6 + 4)$ $= 26 - 23 = 3$ cm
Specific behaviours	
✓ uses scale factor to obtain KL or small pentagon perimeter ✓ correct side length	

The area of $K'L'M'N'O'$ is 30 cm^2.

(c) Determine the area of $KLMNO$. (2 marks)

Solution
Area scale factor is $5 \times 5 = 25$ $\text{Area} = 30 \times 25 = 750$ cm^2
Specific behaviours
✓ indicates correct area scale factor ✓ uses area scale factor to obtain area

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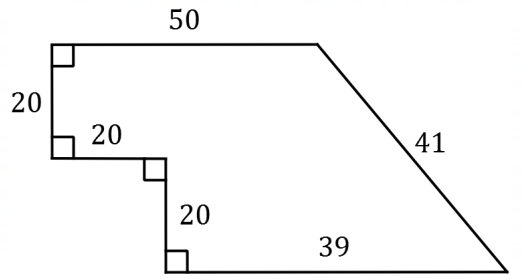
Question 7 **(8 marks)**

The sketch diagram (not drawn to scale) shows a plot of land bounded by six straight sides.

The length of each side is shown in metres.



(a) A fence that costs $\$30$ per metre is to be erected around the perimeter of plot. Determine the total cost of fencing. (3 marks)

Solution
$P = 50 + 41 + 39 + 20 + 20 + 20 = 190 \text{ m}$ $C = 190 \times 30 = \$5700$
Specific behaviours
✓ indicates correct calculation for perimeter ✓ indicates correct calculation for total cost ✓ correct total cost

(b) Once the fence is in place, the plot is to be treated with weedkiller at a rate of 10 mL per square metre. The weedkiller is sold in 3 litre containers. Determine, with justification, the number of containers of weedkiller required. (5 marks)

Solution
Rectangle: $A_R = 50 \times 40 = 2000$ Square: $A_S = 20 \times 20 = 400$ Triangle: $A_T = \frac{1}{2} \times 9 \times 40 = 180$ Total area: $A = 2000 - 400 + 180 = 1780 \text{ m}^2$ Litres of weedkiller: $1780 \times 0.01 = 17.8 \text{ L}$ and so 6 containers will be required as 6x3=18 >17.8
Specific behaviours
✓ correctly partitions plot into suitable composite areas ✓ calculates all composite areas correctly ✓ calculates total area ✓ calculates litres (or mL) of weedkiller required ✓ states number of containers needed with justification

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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